

Hong Kong Mathematics Olympiad (2022/23)
 Finals (Individual – Event 1)

FOR OFFICIAL USE

Score for accuracy	<input type="text"/>	×	Mult. factor for speed	<input type="text"/>	=	<input type="text"/>	Team No.	<input type="text"/>
			+	Bonus score		<input type="text"/>	Time	<input type="text"/>
							Min.	Sec.
						<input type="text"/>		

Unless otherwise stated, all answers should be expressed in numerals in their simplest forms.

除非特別聲明，答案須用數字表達，並化至最簡。

1. If A is the unit digit of 2023^{2024} , find the value of A .

若 A 是 2023^{2024} 的個位數，求 A 的值。

$A =$

2. If B is the number of positive common factors of 336^A and 528^A , find the value of B .

若 B 是 336^A 和 528^A 的正公因數的數量，求 B 的值。

$B =$

3. A 3×3 grid is partially completed as shown below. Fill each square of the grid with a positive integer such that the sum of the three numbers in each row, column, and both main diagonals are equal. Find the value of C .

下圖是一個未完成的九宮格，每一格須填入一個正整數使得每一行、每一列和兩個主對角線上的三個數字總和相等。求 C 的值。

C	16	$2B$
4		

$C =$

4. $\frac{C}{2}$ couples are attending a party, which means that there are C people present. At this party, no one will shake hands repeatedly with the same guest. The party also has the condition that each husband will shake hands with every guest except his own wife, and wives will shake hands with every guest except other wives. D represents the total number of handshakes between the C people at the party. Find the value of D .

有 $\frac{C}{2}$ 對夫婦參加了一個派對，即在派對上共有 C 人。在這個派對上，沒有人會和同一位客人重複地握手。此外，每位丈夫都會和他妻子以外的所有客人握手，而妻子們不會與其他妻子握手，但會和其他客人握手。 D 是在這派對上 C 人之間握手的總數，求 D 的值。

$D =$

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 Finals (Individual – Event 2)

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			+	Bonus score		<input type="text"/>	Time	<input type="text"/>
							Min.	Sec.
						<input type="text"/>		

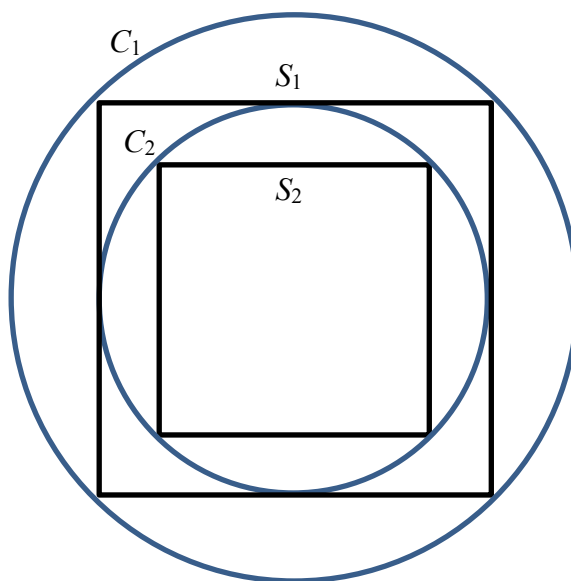
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除非特別聲明，答案須用數字表達，並化至最簡。

1. Find the smallest positive integer α that is divisible by 11 and the sum of its digits is equal to 38.
找出一個能被 11 整除，且各數位之和是 38 的最小正整數 α 。

$\alpha =$

2. Let α' be the last digit of α . A circle C_1 of radius α' circumscribes a square S_1 which inscribes a circle C_2 . C_2 circumscribes square S_2 and so forth indefinitely. Find the area β of the square S_6 .
設 α 的最後一位數字是 α' 。 C_1 是正方形 S_1 的外接圓，它的半徑為 α' ， C_2 是正方形 S_1 的內接圓， C_2 亦是正方形 S_2 的外接圓，如此類推。求正方形 S_6 的面積 β 。



$\beta =$

3. Let $[\beta]$ be the integral part of β . The equation $x^3 + nx^2 + mx + [\beta] = 0$, where m, n are integers, has three integer roots. Suppose that the roots are not all positive, if $\gamma = n - m$, find γ .

設 β 的整數部分是 $[\beta]$ 。若 m, n 為整數，方程 $x^3 + nx^2 + mx + [\beta] = 0$ 有三個整數根。假設這三個根不全是正整數，若 $\gamma = n - m$ ，求 γ 。

$\gamma =$

4. On the xy -plane, a move consists of independently increasing (or decreasing) x -coordinate and y -coordinate by 1 (i.e. moving diagonally). If δ is the number of ways to start from $(0, 0)$, make exactly 12 moves and end at (γ, γ) , find the value of δ .

在 xy 平面上，每一步移動都包含 x 坐標和 y 坐標分別地增加 (或減少) 1 個單位 (即對角線移動)。若 δ 是由 $(0, 0)$ 開始祇行走 12 步後到達 (γ, γ) 的方法，求 δ 。

$\delta =$

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							Min.	Sec.
						<input type="text"/>		

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除非特別聲明，答案須用數字表達，並化至最簡。

1. Given that m and n are positive integers. If $m+n+mn=76$ and $A=m+n$, find the value of A .

已知 m 和 n 均為正整數。如果 $m+n+mn=76$ 及 $A=m+n$ ，求 A 的值。

$A =$

2. If $B = \sqrt{(401)^2 - 100A}$, find the value of B .

如果 $B = \sqrt{(401)^2 - 100A}$ ，求 B 的值。

$B =$

3. The area of the rhombus on the xy -plane with vertices $(B+1, 0)$, $(-B-1, 0)$, $(0, 1)$ and $(0, -1)$ is C square units. Find the value of C .

在 xy 平面上由點 $(B+1, 0)$, $(-B-1, 0)$, $(0, 1)$ 及 $(0, -1)$ 所組成之菱形的面積為 C 平方單位，求 C 的值。

$C =$

4. If D is a positive integer such that $\left(\frac{C}{4} + 56\right)^{\frac{1}{D}} = D$, find the value of D .

如果 D 是正整數且 $\left(\frac{C}{4} + 56\right)^{\frac{1}{D}} = D$ ，求 D 的值。

$D =$

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 Finals (Individual – Event 4)

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Score for accuracy	<input type="text"/>	×	Mult. factor for speed	<input type="text"/>	=	<input type="text"/>
			+	Bonus score		<input type="text"/>
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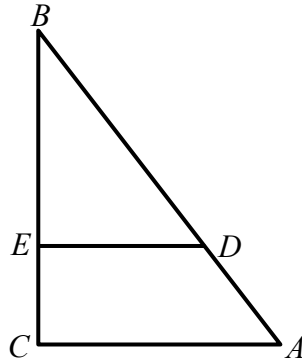
Team No.	<input type="text"/>
Time	<input type="text"/>
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除非特別聲明，答案須用數字表達，並化至最簡。

1. In triangle ABC , $\angle C = 90^\circ$, $DE \perp BC$, $BE = AC$, $BD = \frac{1}{2}$ and $DE + BC = 1$. If $\alpha = 4ED$, find α .

在三角形 ABC 中， $\angle C = 90^\circ$ ， $DE \perp BC$ ， $BE = AC$ ， $BD = \frac{1}{2}$ 及 $DE + BC = 1$ 。如果 $\alpha = 4ED$ ，求 α 。



$\alpha =$

2. If $f(a) = a - 2$, $F(a, b) = b^2 + a + \alpha$ and $\beta = F(3, f(4))$, find β .

若 $f(a) = a - 2$ ，且 $F(a, b) = b^2 + a + \alpha$ 及 $\beta = F(3, f(4))$ ，求 β 。

$\beta =$

3. If the system of equations

$$\begin{cases} x^2 - 3xy + 2y^2 - z^2 = 31 \\ -x^2 + 6yz + 2z^2 = 44 \\ x^2 + xy + \beta \cdot z^2 = 100 \end{cases}$$

has γ sets of integer solutions, find γ .

如果方程組

$$\begin{cases} x^2 - 3xy + 2y^2 - z^2 = 31 \\ -x^2 + 6yz + 2z^2 = 44 \\ x^2 + xy + \beta \cdot z^2 = 100 \end{cases}$$

整數解的個數是 γ ，求 γ 。

$\gamma =$

4. In a triangle ABC , $AB = AC$, $\angle A = 40^\circ + \gamma^\circ$. Point O is inside the triangle ABC with $\angle OBC = \angle OCA$. If $\angle BOC = \delta^\circ$, find δ .

在三角形 ABC 中， $AB = AC$ ， $\angle A = 40^\circ + \gamma^\circ$ 。點 O 在三角形 ABC 內且 $\angle OBC = \angle OCA$ 。如果 $\angle BOC = \delta^\circ$ ，求 δ 。

$\delta =$